Section 1.1: Differential equations and mathematical models

New vocabulary:

- differential equation
- Order of a d.e.
- Solution of a d.e.
- Ordinary and partial d.e.'s

one variable

>1 variable.

- Initial condition, initial value problem

An equation like $\frac{d^3P}{dt^3} = P^5 + t$

A solution is a function $P(t)$

The order is the highest derivative that occurs

This one has order 3

Initial condition is like $P(0) = 23$

D.E. + I.C = an initial value problem.

1.1 question 20: a) Verify that y(x) satisfies the given d.e.

b) Find a value of C so that y(x) satisfies the given initial condition.

c) Sketch several typical solutions of the d.e. and highlight the one that satisfies the given initial condition.

$\underbrace{y' = x-y;}_{\text{d.e.}}$ $\underbrace{y(x) = Ce^{-x} + x - 1;}_{\text{solution}}$ $\underbrace{y(0) = 1.}_{\text{I.C.}}$

a) $y' = -Ce^{-x} + 1$, $x - y = x - Ce^{-x} - x + 1$

These are equal.

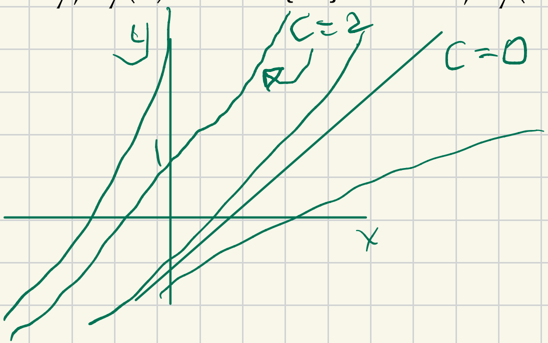
b) $y(0) = Ce^{0} + 0 - 1 = C - 1 = 1$,

so $C = 2$.

1.1 question 20: Verify that $y(x)$ satisfies the given d.e.
Find a value of C so that $y(x)$ satisfies the given initial condition.
c) Sketch several typical solutions of the d.e. and highlight the one that satisfies the given initial condition.

$y' = x-y$; $y(x) = Ce^{-x} + x - 1$; $y(0) = 1$.

c)



$C = 0: \; y = x - 1$

Class Activity!!!

If $y(0) = 10$, what is C ?

a. e^{10}

b. 9

c. 10

d. 11 ✓

e. None of the above.

Some equations in the text.

Example 3: Newton's law of cooling where the body temperature is $T(t)$ and the ambient temperature is A.

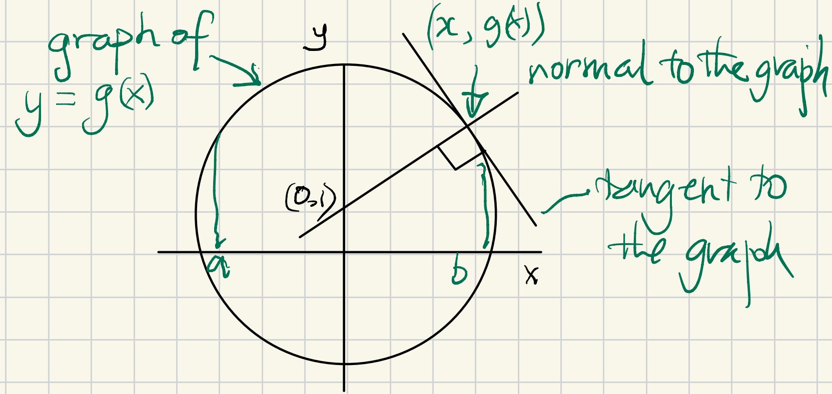
$$\frac{dT}{dt} = k(T - A)$$

We don't need to know Example 4: Torricelli's law.

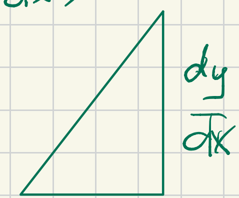
Example 5: the size of a population $P(t)$ with constant birth and death rates.

$$\frac{dP}{dt} = kP$$

Section 1.1 question 29.
Write a differential equation $dy/dx = f(x,y)$ having a function g as one of its solutions, where g is described by:
Every straight line normal to the graph of g passes through the point $(0,1)$.



We find a tangent vector to the graph. The vector $\left(1, \frac{dy}{dx}\right)$ has slope $\frac{dy}{dx}$. This points in the tangent direction.



The normal line passes through $(0,1)$ and $(x, g(x))$ so $(x, g(x)-1)$ points in the normal direction.

Equation: $(1, y') \cdot (x, y-1) = 0$

$x + y'(y-1) = 0$ is the required d.e.